NAME:______________________

ECEN 5713 System Theory
Spring 1997
Midterm Exam #2

I, ____________________, promise that I won't seek any help from others. And I won't discuss with anyone else. ________________________________
signature

Classification of Systems (20%)

Problem 1a) Consider a single-variable system whose input and output are related by

$$y(t) = \begin{cases} \dfrac{u^2(t)}{u(t-1)} & \text{if } u(t-1) \neq 0 \\ 0 & \text{if } u(t-1) = 0 \end{cases}$$

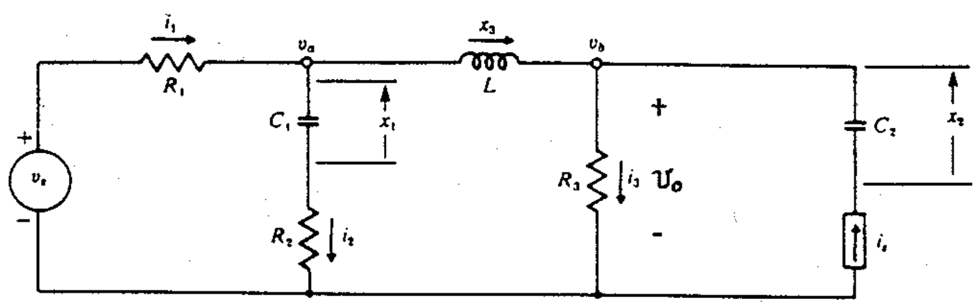
for all t. Is this system linear ? causal ? time-invariant ?
Problem 1b) Consider a relaxed system whose input and output are related by

$$y(t) = \begin{cases} u(t) & \text{for } t \leq \alpha \\ 0 & \text{for } t > \alpha \end{cases}$$

for any u, where α is a fixed constant. Is this system linear ? causal ? time-invariant ?

System Representation (20%)

Problem 2 Find all three representations (i.e., input-output operator, transfer function, and state space equations) of the following RLC circuit,



Linearization (20%)

Problem 3 A nonlinear system is given by

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, u_1, u_2) \\ f_2(x_1, x_2, u_1, u_2) \end{bmatrix} = \begin{bmatrix} 3 + \ln(1 + x_1 x_2) + \ln(1 - 5x_1) + \sin^2(5u_1) \\ x_1(2 + x_2)^2 - \cos(5x_2) - e^{2u_2} \end{bmatrix}.$$

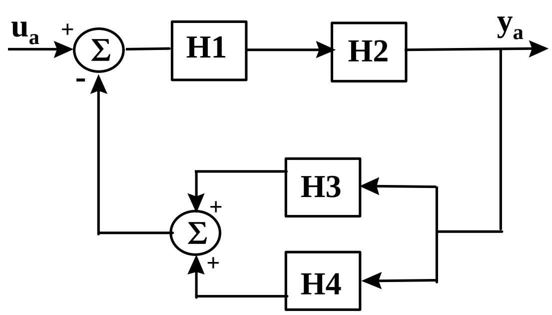
Note that $x = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ is an equilibrium point at $u = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$. *Linearize* the system about the equilibrium point. To improve the accuracy, approximate up to the *second order* in the linearization process in Taylor series expansion. Find the linearized system (my be not in the form of $\{A, B, C, D\}$).

<u>Realization</u> (20%, do both)

<u>Problem 4a</u>) Find an irreducible (i.e., minimal) controllable canonical form realization (i.e., its simulation diagram and state space equations) for the following system,

$$H(s) = \begin{bmatrix} \dfrac{2s+3}{s^3+4s^2+5s+2} \\ \\ \dfrac{s^2+2s+2}{s^4+3s^3+3s^2+s} \end{bmatrix} \text{(hint: A is } 5\times 5\text{).}$$

<u>Problem 4b</u>) Find the $\{A, B, C, D\}$ matrices of the composite interconnected system given below,



where $\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + B\, u_a; \quad y_a = C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + D\, u_a$ and

$H_i \equiv \{A_i, B_i, C_i, D_i\}, i = 1,2,3,4$ (hint: you may stop at the temporary variables which are functions of $\{A_i, B_i, C_i, D_i\}$, i = 1, 2, 3, 4).

<u>Linear Algebra</u> (20%)

<u>Problem 5a</u>) Given the set $\{a, b\}$ with $a \neq b$. Define rules of addition and multiplication such that $\{a, b\}$ forms a field. What are the zero and unity elements in the field ?

<u>Problem 5b</u>) Let $E = [e_1 \quad e_2 \quad \cdots \quad e_n]^T$ be a column vector of error in a multivariable control system. Show that the sume of the squares of the error can be written in several forms, $e_1^2 + e_2^2 + \cdots + e_n^2 = E^T E = \mathrm{Tr}(EE^T)$.

HOW LONG YOU HAVE SPENT ON THIS EXAM ?